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PERCEIVED DIFFICULTY IN MATHEMATICS EDUCATION

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Mathematics education researchers have widely studied the possible characteristics that influence the difficulty of solving a task. This study focuses on the perspective of students and teachers and their perceived difficulty, which have not been equally addressed. Difficulty in an absolute sense and perceived difficulty are different but intertwined: task characteristics can shape students' and teachers' perceptions, and these perceptions of difficulty can affect the solver's behaviour, influencing the accuracy of the response. Our study focuses on the factors influencing secondary school students' and teachers' evaluations of task difficulty. We describe the process that led to the development of macro-categories for perceived difficulty, highlighting both the study's limitations and potential directions for future research.

INTRODUCTION

The challenge of difficulty in mathematics has been extensively examined within the field of Mathematics Education. Difficulties can be classified into different categories depending on the focus: difficulties related to mathematics as a discipline or difficulties related to students in terms of deficits or their specific relationship with mathematics (e.g. Zan et al., 2006).

However, the issue of *perceived difficulty* (PD) and its underlying causes have not received equal attention. In particular, PD and its causes have not been addressed in the same way. Within the field of mathematics education research, there is no shared definition of PD, and few studies have explored the factors related to this issue (Eccles & Wigfield, 2020; Doz et al., 2023). It is therefore necessary to first clarify what is the meaning of PD. In this research, we consider PD in relation to the process of solving a mathematical task. During the process of solving a task, a student may encounter various difficulties, which can stem from both the student's own characteristics (such as abilities, knowledge, beliefs, and attitudes) and the characteristics of the task itself (such as the wording or the mathematical content involved). These features of the mathematical task are also the ones that can influence the student's perception of the task and, consequently, their PD. Therefore, while closely related, difficulty and PD are two distinct aspects (Spagnolo & Saccoletto, 2023). We argue that one of the main differences is that PD can influence students' behaviour when approaching a mathematical task. In addition, this research examines the difficulties associated with mathematical tasks, with particular emphasis on the PD experienced by students and teachers *after* facing a task. The literature differentiates between difficulty before and after a task has been attempted or solved, a distinction that is crucial for understanding

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also PD. However, this study specifically focuses on PD after students and teachers have faced the task.

The research began in 2021 during the pandemic period, with a study presented at ICME14 published in an extended paper (Saccoletto & Spagnolo, 2022). Here, the aim is to highlight the process that led to the identification of the factors, the development of categories for analyzing PD, and the description of these categories. Early phases of this study have been presented at international conferences such as ICME14, CERME13, MAVI29, ICME15, PME47, MAVI30, and CERME14. Each of these studies provided key elements to the identification of these macro-categories.

THEORETICAL BACKGROUND

In Mathematics Education, there is no shared definition of PD of a task, although it is generally recognized that PD differs from a task's inherent difficulty, which is often determined retrospectively based on students' results (Mehrens & Lehmann, 1991). Instead, research in cognitive psychology has analysed the theme of subjective difficulty and its perception since the 1990s, resulting in various definitions and expressions to characterise the concept (Eccles & Wigfield, 2020; Doz et al., 2023).

First of all, generally speaking, the perception of the difficulty of a task is sometimes considered dependent on the cognitive load that the task implies (Hellmann & Nuckles, 2013). Sometimes, PD has been regarded as a different manifestation of self-efficacy rather than a complementary aspect to it, but this unification does not seem justifiable (Eccles & Wigfield, 2020). One of the concepts closer to the PD as we mean it is the "feeling of difficulty" (FOD), progressively defined by Efklides and colleagues in different studies between the 1990s and 2011. Initially, the FOD was simply described as one of "the closest estimates of feelings of difficulty" (Efklides et al., 1998). The authors acknowledged the impossibility of directly measuring feelings but considered it possible instead to have people rate the difficulty of a task. In a second moment, Efklides and colleagues analysed the various causes of FOD, defining it as a "metacognitive experience that monitors cognitive processing as it takes place" (Efklides & Touroutoglou, 2010, p. 272). FOD is related to PD, yet they are conceptually distinct, as FOD has an "experiential nature" while PD arises from metacognitive judgments based on a conscious recall of self-knowledge and task-related information. Obviously, individuals may draw upon their metacognitive knowledge about the task and themselves to interpret the feelings they experience (Efklides & Touroutoglou, 2010). Taking into account the differences, in this paper we refer to PD, considering the characteristics that make it similar to FOD and adapting them to our context, that is Mathematical Education research. It is worth mentioning the synthesis presented by Doz et al. (2023), stating that the feeling of task difficulty is metacognitive in nature because it derives from monitoring the activity of a developing task processing and the consciousness of this process influences self-regulation, effort, affect, and strategy use.

In order to define the aforementioned macro-categories that describes PD, we draw upon theories and constructs used in Mathematical Education research. Firstly, we considered as a reference to analyse students' answers the *multidimensional model of attitude* (Di Martino & Zan, 2010), which takes into account the perceived competence, the emotional aspects and the vision of mathematics. In addition to that, the aspect of metacognition was also considered, as it is connected to students' decision-making process when addressing a problem, a process influenced by personal beliefs and values (Radmehr & Drake, 2017). We also considered the concept of *expert blind spot* (Nathan & Petrosino, 2003), an expression used to identify teachers' lack of awareness of students' understanding and difficulties associated with great content knowledge. Moreover, the experts may even be unaware of this blind spot deriving from expertise itself. Referring to the concepts of pedagogical content knowledge and subject matter knowledge (Ball et al., 2008), the authors argue that teachers showing an expert blind spot may have both of them but, when applying this knowledge to "a specific area of mathematics, such as algebra instruction, those bodies of knowledge come into conflict" (Nathan & Petrosino, 2003). The concept of expert blind spot can be another hypothesis to explain the different PD that students and teachers experience.

METHODOLOGY

This section will describe the different phases that allowed for the development and definition of the macro-categories. These macro-categories allow to describe factors influencing students' and teachers' PD. Specifically, the first phase, presented at ICME14 (Saccoletto & Spagnolo, 2022), aimed to investigate the correlation between PD and students' ability to answer the task correctly. Then, the second phase, presented at CERME13 (Spagnolo & Saccoletto, 2023), ICME15 and PME47 (Nicchiotti & Spagnolo, 2024), made it possible to clarify the categories that were used to classify the factors influencing students' PD. These categories were determined using Constructive Grounded Theory (CGT), employing an inductive method and directly drawing on the data collected (Glaser & Strauss, 1967). Phases 3 (presented at MAVI29, MAVI30, and CERME14) comparing the factors that influence PD of teachers. All the research phases involved secondary school students and teachers.

First phase: correlation between PD and students' ability to answer the task

In this first qualitative phase, 79 Italian students from a high school from humanities curriculum were involved: two grade 9 classes and two grade 10 classes. The students completed an online questionnaire, followed by in-depth interviews. The questionnaire was administered via Google Forms (using the classroom computers), and a researcher was present during the administration. The questionnaire included two argumentative algebraic tasks related to literal calculus (Figure 1). Specifically, Task 1 is a multiple-choice question that requires the recognition of a correct argument, while Task 2 is an open-ended question that requires the production of an argument; both tasks were selected from previous national INVALSI tests, as they are statistically validated (Lazarsfeld, 1958).

Task 1

Antonio states that «4n-1 is always a multiple of 3».
Is Antonio right?
In the following table select the only argumentation that justifies the right answer

Antonio is right...		Antonio is not right...	
A. ☐	because 4n-1=3n	C. ☐	because 4n-1 is always odd
B. ☐	because if n=4 then 4n-1=15	D. ☐	because if n=3 then 4n-1=11

Task 2

Marco states that, for every natural number n greater than 0, $n^2 + n + 1$ is a prime number.
Is Marco right?

Choose one of the two answers and complete the sentence.

☐ Marco is right because ....

☐ Marco is not right because ....

Figure 1: Task 1 and Task 2 (original texts from www.gestinv.it., translation provided by the authors).

After solving each task, students evaluate its difficulty (explaining the reasons for their evaluation) and answer other questions related to the PD of the specific task. Finally, the last section of the questionnaire was more general, with questions aimed at clarifying the underlying factors that might influence the PD. The interviews were conducted remotely via Google Classroom (Meet applet) with the aim of helping to categorize some of the responses provided by the students. Among the works addressing the need to develop theoretical frameworks on affect, we specifically refer to the study by Di Martino and Zan (2010) on attitude, as we recognized some analogies with their study in reading and analyzing the responses of our students.

Second phase: development and generalization of the categories

The second phase of the study (Spagnolo & Saccoletto, 2023) involved 148 students from schools of different types (five grade 9 classes, and two grade 10 classes). The students completed the same online questionnaire (described in the previous section), followed by in-depth, face-to-face interviews aimed at facilitating the categorization of the responses provided by the students. These responses were categorized using CGT (Charmaz, 1994), with particular attention to the open-ended questions in the questionnaire. The categories of analysis were developed from the specific cases, but conclusions were drawn from all the database (which consisted of descriptive data provided by the students). The categories that emerged from the analysis were compared with the categories of the theoretical framework proposed by Di Martino and Zan (2010), highlighting differences with PD. To analyze and emphasize these differences, focus groups were conducted with the same students. Significant regularities in the data collection proved valuable insight in constructing the categories of analysis.

Third phase: comparison factors influencing PD of students and teachers

Finally, the third phase of the study (Nicchiotti & Spagnolo, 2024) involved teachers, specifically 49 secondary mathematics teachers. The individual characteristics of the teachers varied considerably, particularly in terms of the type of school and curriculum in which they teach, years of experience, and educational background. As in the previous phases, the teachers completed the online questionnaire (with respect to the same tasks evaluated by the students), followed by in-depth interviews conducted via Google Meet. The qualitative analysis of the data was grounded in the factors characterizing the students' PD, which were compared with factors expressed by the teachers.

RESULTS AND DISCUSSION

We used Excel to calculate mean values and represent the data graphically, while NVIVO 12 was used to conduct the qualitative analysis of the responses expressed by students and teachers.

First phase: correlation between PD and students' ability to answer the task

Analysis highlights that student's PD does not appear to be related to the ability to answer a question correctly but seems to encompass broader metacognitive and self-perception factors. From the qualitative analysis of the questionnaire, it also emerges that there is no correspondence between the PD level selected during the evaluation of individual tasks (in response to "On a scale from 1 to 10, how difficult did you find Task 1/Task 2?") and the level selected during the comparison of the two tasks (in response to "Compare the two tasks. Which one did you find more difficult?"). In particular, students expresses different level of PD in relation to a single task or in the comparison of multiple tasks (see Figure 1 for Task 1 and 2). Student interviews suggest that this choice is influenced by factors related to attitudes and beliefs. In our view, this may be evidence of students' difficulty in evaluating a task or may indicate that students consider different factors when assessing individual tasks versus comparing them.

Second phase: development and generalization of the categories

The aim of this phase was to further investigate the motivations that lead students to choose one level of difficulty over another. An initial analysis highlighted several categories related to the main aspects mentioned by students in their responses. We considered these categories, re-examined the responses, and worked to unify, compare, and clarify the main aspects. The analysis provide five main macro-categories (Spagnolo & Saccoletto, 2023): *Resolution Strategy*, *Capabilities and Experience*, *Emotions*, *Task Formulation*, and *Self Consideration*.

Resolution Strategy category, grouped responses where students explicitly referred to the type of strategy or process they believed was necessary to solve the task. The second category, *Capabilities and Experience*, is the most prevalent and pertains to responses that refer to the abilities or competencies students perceive and to previous experiences that influence their PD toward the task. This category also includes students who explicitly state that they are unfamiliar with this type of task, reinforcing the idea that a problem is easier if it resembles something already known. Additionally, this category includes responses referring to what students can (cannot) do or (do not) know. The focus in these responses is on students' self-perception (either in general or regarding the tasks). The *Emotions* category refers to students explicitly considering their emotions when justifying the level of PD chosen. The fourth category, *Task formulation*, represents considerations about the task's structure, especially in relation to the wording. The fifth category, *Self Considerations*, refers to students' personal reflections on their success in mathematics.

The categories emerging from the analysis help clarify the main aspects involved when a student expresses her or his PD in relation to mathematical tasks. These categories are not exclusive, and some responses may be classified under more than one category.

We believe that these five categories can be linked to the attitude construct in the sense of Di Martino and Zan (2010). Additionally, the responses to the questionnaire can be related to students' view of mathematics: some students explicitly describe the methods they consider necessary to solve tasks. The Self Considerations category contains responses that express what students believe is necessary for success in mathematics. Finally, even in our case, students refer, more or less explicitly, to their perceived competencies in solving tasks, while also expressing ideas about their perceived knowledge and skills. However, when assigning a PD level to a task, students seem to be influenced by factors more directly related to the task (such as elements in the wording), by factors linked to their attitude or emotions, and by metacognitive aspects (such as the inability to assess their own competencies, knowledge, and skills, or personal reflections on how to improve their performance).

The analysis of students' ratings and answers allowed also to state that there seem to be some differences between boys' and girls' PD. Boys tend to evaluate mathematical tasks as easier than girls do, even regardless of their actual performance solving them (Nicchiotti & Spagnolo, 2024).

Third phase: comparison factors influencing PD of students and teachers

This phase provides an initial overview of the factors influencing teachers' PD, comparing them with those influencing students' PD. The macro-categories defined in Spagnolo & Saccoletto (2023) are useful for analyzing teachers' responses. The factors influencing the PD of both teachers and students are similar, but they have different proportion. Generally, teachers appear to be aware of the reasons behind PD, but sometimes they underestimate them and seem to overlook the emotional aspects related to difficulty. This tendency aligns with the notion that experts, due to their deep knowledge, may struggle to recognize the challenges faced by learners, as their expertise can create a disconnect between their understanding and students' perceptions. In this context, teachers seem to focus more on objective factors, such as the form and content of the task, when assessing its PD. In contrast, students also consider subjective elements, such as experience or self-perception. This may reflect a broader cognitive gap that the expert blind spot theory helps to explain.

CONCLUDING REMARKS

This research gives a first insight into the PD and the factors characterizing it, comparing students' and teachers' perspectives. PD has a strong affective connotation, as it entails beliefs, self-efficacy and emotions. The main factors used to describe PD of students and teachers in relation to a mathematical task are summarized in the five macro-categories: Resolution Strategy, Capabilities and Experiences, Emotions, Task Formulation, and Self-consideration. Figure 2 shows a visual representation of PD. The eye, symbolizing the personal perspective, is surrounded by the five macro-categories influencing PD, which intersect at the center of the eye. This highlights that the macro-categories are not mutually exclusive, but may overlap (meaning that a student or teacher may consider more than one category when explaining why they perceive a task as difficult or easy).

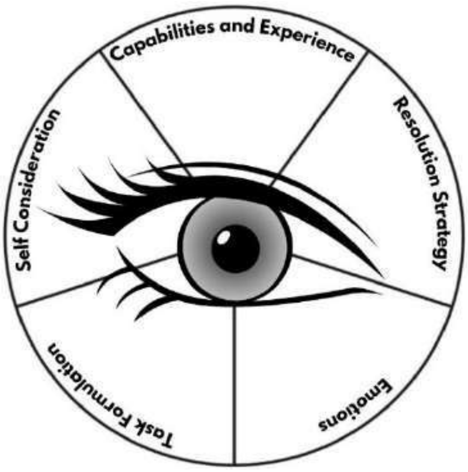

Figure 2: Visual representation of macro-categories that describe PD.

The five macro-categories based on the factors influencing students' PD *after* solving mathematical tasks are essential for providing a definition of PD. However, the validity of these findings could be strengthened considering expanding the study working with a bigger sample and more tasks, of different types and regarding other math topics. Further study will examine the phenomenon from a quantitative perspective.

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PME 48 2025

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